Computation of Edge-Edge-Edge Events Based on Conicoid Theory for 3-D Object Recognition

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Abstract: The availability of a good viewpoint space partition is crucial in three dimensional (3-D) object recognition on the approach of aspect graph. There are two important events depicted by the aspect graph approach, edge-edge-edge (EEE) events and edge-vertex (EV) events. This paper presents an algorithm to compute EEE events by characteristic analysis based on conicoid theory, in contrast to current algorithms that focus too much on EV events and often overlook the importance of EEE events. Also, the paper provides a standard flowchart for the viewpoint space partitioning based on aspect graph theory that makes it suitable for perspective models. The partitioning result best demonstrates the algorithm’s efficiency with more valuable viewpoints found with the help of EEE events, which can definitely help to achieve high recognition rate for 3-D object recognition.

Key words: edge-edge-edge (EEE) event; aspect graph; viewpoint space partition; critical events; three dimensional (3-D) object recognition

Introduction

Aspect graphs have been used for enhanced three dimensional (3-D) object recognition since Gigué et al.[1] proposed the idea in 1991. Aspect graphs have been proven effective by Schiffenbauer[2] and by Tambouratzis and Wright[3]. In this approach, the viewpoint space is partitioned into finite areas of sufficient information to represent the complete topology of an object. Since the projections in each area are identical in topological structure; then, a prototypical projection can be generated for each area. Thus, the problem of 3-D object recognition can be simplified to a sequence matching algorithm of two dimensional (2-D) projections.

The two critical events in viewpoint space partition are edge-vertex (EV) events and edge-edge-edge (EEE) events. The geometrical analysis of the locus of the accidental viewpoints for EV events has long been thought to be simpler than that of the EEE events. Thus, many current aspect graph algorithms for 3-D object recognition applications such as vertex clustering[4], triangle contraction confined by envelopes[5], and edge collapse[6-10], either only take EV events into account, or pay little attention to EEE events in light of the significant calculation complexity[10].

However, contrary to the current algorithms, the characteristics of the viewpoint space partitions by EEE events based on analytical geometry, specifically, conicoid theory, significantly reduce the calculation complexity of the EEE events. This practical method proposes a meaningful classification of EEE events, which changes our understanding of aspect graphs. This method has been applied to three kinds of plane models, a car model, and a blimp model to show that it can effectively carry out space partition with more abundant information. The results provide a foundation
for a heuristic discussion of the physical nature of EEE events.

1 Analytical Geometry Analysis of EEE Events

Aspect graph theory was thoroughly discussed by Schiffenbauer[2], who pointed out that there are altogether seven types of visual events, point edge, triplet, tangent cross, cusp, swallow, beak, and lip types. To partition the viewpoints on the basis of EV and EEE events, only the first two types are considered. Their mathematical descriptions are

Point edge: \[ I(x_0,y_0) = 0 \] (1)

Triplet: \[ I(x_0,y_0) = I(x_0,y_0) = I(x_0,y_0) \] (2)

EEE events have long been computed in the same way as EV events, by partitioning the viewpoint space by the sign of the critical events’ equation. This method is effective to some extent; however, the symmetric results obtained with this method as depicted in Fig. 1 suggest that from a solid geometry viewpoint, any aspect graph cannot cover more than half of the sphere; thus, such a method is not rational, or at least not necessary.

These observations of previous results give rise to an approach to compute EEE events. Since the EEE event equation can be expressed as a conicoid, the EEE event can be identified from the properties of a conicoid.

2 Simplification and Proof of the EEE Events Equation

Let the mutually skew edges which form the EEE event be \( e_1, e_2, \) and \( e_3 \), in which \( e_i = (a_i, b_i) \). Let \( d_i = b_i - a_i \) be the \( e_i \) direction vectors and let \( l_i \) be the line on which \( e_i \) lies. Assume that \( p \) is an arbitrary viewpoint. Gigus et al.[11] pointed out that the accidental viewpoint lies on the surface of

\[ (p-a_1) \times d_1 \times (p-a_2) \times d_2 \times (p-a_3) \times d_3 = 0 \] (3)

To deduce the EEE event equation, assume that \( p=(x,y,z), a_i=(x_{i0},y_{i0},z_{i0}), b_i=(x_{i0},y_{i0},z_{i0}), \) \( (i=1, 2, 3, \text{ same below}) \)

Thus, \[ d_i = (d_{i0}, d_{iy}, d_{iz}) = (a_1-a_{i0}, a_2-a_{i0}, a_3-a_{i0}) \]

\[ c_i = (p-a_i) \times d_i = [(y-y_{i0})d_{iz} - (z-z_{i0})d_{iy}]x + \\
(z-z_{i0})d_{ix} - (x-x_{i0})d_{iz}]y + (x-x_{i0})d_{iy} - (y-y_{i0})d_{iz}z \] (4)

Expanding the coefficients and substituting for the variables gives

\[ c_i = (e_{00}y + e_{01}z + e_{02})x + (e_{20}x + e_{21}z + e_{22})y + (e_{00}x + e_{01}z + e_{02})z \] (5)

Each \( e_{ij} \) is a constant in the expansions of Eq. (4). Substituting Eq. (5) into Eq. (3) gets

\[ (e_{00}y + e_{01}z + e_{02})(e_{10}x + e_{11}z + e_{12}) + (e_{20}y + e_{21}z + e_{22})(e_{10}x + e_{11}z + e_{12}) = \\
(e_{00}y + e_{01}z + e_{02})(e_{20}x + e_{21}z + e_{22}) + (e_{20}y + e_{21}z + e_{22})(e_{00}x + e_{01}z + e_{02}) \]

\[ + (e_{00}y + e_{01}z + e_{02})(e_{20}x + e_{21}z + e_{22}) + (e_{20}y + e_{21}z + e_{22})(e_{00}x + e_{01}z + e_{02}) = 0 \] (6)

Considering the coefficients of the cubic terms in Eq. (6)’s expansion, for example, the coefficient of \( x^3y \) is

\[ e_{03}e_{13}e_{26} - e_{26}e_{32}e_{16} + e_{26}e_{33}e_{16} - e_{26}e_{33}e_{26} + \\
e_{03}e_{13}e_{26} - e_{26}e_{32}e_{16} = \\
(\beta_{32} - a_{33}) \times (\beta_{33} - a_{13}) \times (\beta_{33} - a_{13}) - \\
(\beta_{32} - a_{33}) \times (\beta_{33} - a_{13}) \times (\beta_{33} - a_{13}) + \\
(\beta_{32} - a_{33}) \times (\beta_{33} - a_{13}) \times (\beta_{33} - a_{13}) + \\
(\beta_{32} - a_{33}) \times (\beta_{33} - a_{13}) \times (\beta_{33} - a_{13}) - \\
(\beta_{32} - a_{33}) \times (\beta_{33} - a_{13}) \times (\beta_{33} - a_{13}) \times (\beta_{33} - a_{13}) = \\
(\beta_{32} - a_{33}) \times (\beta_{33} - a_{13}) \times (\beta_{33} - a_{13}) \times (\beta_{33} - a_{13}) + \\
(\beta_{32} - a_{33}) \times (\beta_{33} - a_{13}) \times (\beta_{33} - a_{13}) \times (\beta_{33} - a_{13}) = \\
(\beta_{32} - a_{33}) \times (\beta_{33} - a_{13}) \times (\beta_{33} - a_{13}) \times (\beta_{33} - a_{13}) = 0 \] (7)

Thus, the general equation for the EEE critical region should be a quadratic equation,

\[ a_1x^2 + a_2y^2 + a_3z^2 + a_4xy + a_5xz + a_6yz + \\
b_1x + b_2y + b_3z + c = 0 \] (8)
A useful classification is obtained from an orthogonal transformation of Eq. (8), which is best depicted by conicoid theory.

\[ a_1x^2 + a_2y^2 + a_3z^2 + 2a_4xy + 2a_5xz + 2a_6yz = \]

where \( a, a, a, a, a, a \) are the eigenvalues of A. Let A be a 4x4 matrix.

Applying an orthogonal transformation to the coordinates gives

\[ [x' y' z'] = Q[x y z]^T \] (10)

Then, the standard equation is obtained from the original equation, Eq. (8),

\[ \lambda_1 x'^2 + \lambda_2 y'^2 + \lambda_3 z'^2 + b_1'x' + b_2'y' + b_3'z' + c = 0 \] (12)

which can be used in computing EEE events for partitioning the critical regions by conicoid theory.

3 Classification of EEE Events by Conicoid Theory

Theoretically, there are 17 different conditions for a ternary quadratic equation. These conditions are a real ellipsoid, an imaginary ellipsoid, a hyperboloid of one sheet, a hyperboloid of two sheets, a real quadric cone, an imaginary quadric cone, an elliptic paraboloid, a hyperbolic paraboloid, a real elliptic cylinder, an imaginary elliptic cylinder, a hyperbolic cylinder, two real intersecting planes, two imaginary intersecting planes, a parabolic cylinder, two real parallel planes, two imaginary parallel planes, and two coincident planes. For practical reasons, this analysis only takes two conditions into consideration for the following reasons:

If A is not a full rank matrix, the conicoid can be degraded to a plane. However, the results of Matlab experiments demonstrate that there is no eigenvalue which is exactly equal to zero since errors always exist in numerical calculations. The analysis could assume that if one eigenvalue is much smaller than the others, let it be zero. However, the results are far from satisfactory in view of the standard partitioning method, while without this assumption, the results are acceptable. Therefore, we have enough evidence to only consider the conditions in which A is a full rank matrix.

From analytical geometry theory, if A is a full rank matrix, there are altogether six conditions corresponding to A. Rearranging Eq. (11) to clearly analyze the problem,

\[ \lambda_1 \left( x' + \frac{b_1'}{2\lambda_1} \right)^2 + \lambda_2 \left( y' + \frac{b_2'}{2\lambda_2} \right)^2 + \lambda_3 \left( z' + \frac{b_3'}{2\lambda_3} \right)^2 + c - \frac{b_1'^2}{4\lambda_1} - \frac{b_2'^2}{4\lambda_2} - \frac{b_3'^2}{4\lambda_3} = 0 \] (13)

Let S be

\[ c - \frac{b_1'^2}{4\lambda_1} - \frac{b_2'^2}{4\lambda_2} - \frac{b_3'^2}{4\lambda_3} \] The equation can then be simplified to

\[ \lambda_1 \left( x' + \frac{b_1'}{2\lambda_1} \right)^2 + \lambda_2 \left( y' + \frac{b_2'}{2\lambda_2} \right)^2 + \lambda_3 \left( z' + \frac{b_3'}{2\lambda_3} \right)^2 + S = 0 \] (14)

Thus, the classifications can then be listed in Table 1.

<table>
<thead>
<tr>
<th>Eigenvalues and S</th>
<th>Surface shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_1, \lambda_2, \lambda_3 &gt; 0 ), ( S &lt; 0 )</td>
<td>Real ellipsoid</td>
</tr>
<tr>
<td>( \lambda_1, \lambda_2, \lambda_3 &gt; 0 ), ( S &gt; 0 )</td>
<td>Imaginary ellipsoid</td>
</tr>
<tr>
<td>( \lambda_1 &gt; 0 ), ( \lambda_2, \lambda_3 &gt; 0 ), ( S = 0 )</td>
<td>Single point</td>
</tr>
<tr>
<td>( \lambda_1, \lambda_2 &gt; 0 ), ( \lambda_3 &lt; 0 ), ( S &gt; 0 )</td>
<td>Hyperboloid of two sheets</td>
</tr>
<tr>
<td>( \lambda_1, \lambda_2 &gt; 0 ), ( \lambda_3 &lt; 0 ), ( S &lt; 0 )</td>
<td>Hyperboloid of one sheet</td>
</tr>
<tr>
<td>( \lambda_1, \lambda_2 &gt; 0 ), ( \lambda_3 &lt; 0 ), ( S = 0 )</td>
<td>Real quadric cone</td>
</tr>
</tbody>
</table>

3.1 Classification perspective

From the classifications in Table 1, the imaginary ellipsoid and single point are of little practical use for partitioning the critical regions. Also, the real quadric cone requires \( S=0 \), which is almost impossible in numerical analysis, so the analysis needs only to consider whether the surface shape is a real ellipsoid (simply called an ellipsoid when no confusion arises), a
hyperboloid of two sheets or a hyperboloid of one sheet.

However, careful examination of test results for relatively simple objects (very simple objects such as ellipsoids or cones will not lead to EEE events) has shown that when the surface shape is an ellipsoid, the objects are spherically symmetric to some extent and, thus, the EEE events occurring in such objects are, obviously, not of interest, at least for partitioning the critical regions. Furthermore, the positioning relationship between an ellipsoid and a Gauss sphere is not well defined. Therefore, there are only two conditions left.

The hyperboloid is commonly used to demonstrate how the viewpoints should be distributed. Although its mathematical descriptions are not yet complete, it does reflect some physical characteristics of the object, which EV events cannot do. Therefore, the following algorithm was developed to compute the EEE event.

3.2 EEE event algorithm based on classifications

A specific EEE event whose corresponding equation can be simplified to a one- or two-sheet hyperboloid will partition the Gauss sphere into three critical regions.

Regardless of the sign of $S$, if there are two positive eigenvalues and the viewpoints satisfy the following equation,

$$
\lambda_1 \left( x^2 + \frac{b_1^2}{2\lambda_1} \right) + \lambda_2 \left( y^2 + \frac{b_2^2}{2\lambda_2} \right) + \lambda_3 \left( z^2 + \frac{b_3^2}{2\lambda_3} \right) + S > 0
$$

then they should be distributed in the middle region. Otherwise, they should be distributed into the two symmetric regions. However, when there is only one positive eigenvalue, the two conditions exchange their results. Therefore, the key to compute EEE events is how to partition the two symmetric regions.

Equation (10) is used to obtain the original coordinates of the positive direction in the new coordinate system. Then, the normal direction of the symmetric surface of the hyperboloid can be determined from Eqs. (10) and (14). The inner product of line segment starting at the center of the hyperboloid and ending on the surface of the two symmetric regions and the normal of the symmetric surface can be used to differentiate the two regions. Since the two regions are not adjacent to each other, the sign of the inner product is sufficient to partition the regions, which also ensures that even a little error in the computation will not affect the result.

This analysis leads to the following process for computing EEE events:

1. Obtain the coefficients of the EEE equation from the points’ coordinates on the three edges which include the EEE event.
2. Compute the eigenvalues of $A$ in Eq. (9) and the transform matrix $Q$.
3. Determine the surface type through the orthogonal transform.
4. If the surface is a hyperboloid, store the EEE event and compute the center of the surface, the normal vertex, and the number of positive eigenvalues.
5. The computation of all the EEE events shall include all viewpoints which can be distributed into three distinct critical regions according to the principles discussed in Section 3.1.

Thus, pruning and clustering will give the actual viewpoint space partitions.

4 Results

The algorithm was used to construct the aspect graph database for five objects:

1. Airship: blimp;
2. Airplane: F4, F16, F117;
3. Automobile: racecar.

These five objects were all to be in the sky, so their viewpoint spaces were all typical spheres. The database for each object includes a viewpoint space file (VPS, the positioning relationships between viewpoints or regions need only one index and three floating points for each region) and the projection of every prototypical viewpoint in the VPS file.

Figure 2 shows the distributions of all the viewpoints for the blimp considering only EV events and considering both EV and EEE events. The number of viewpoints is increased significantly by considering both types of events. The additional viewpoints are not only essential but also related to the nature of the EEE events.

The viewpoint groups can be divided as shown by the sphere in Fig. 3.
the EV events. The other two groups are quite different. The new approach gives almost 50% more viewpoints than the original approach. For the blimp, these additional results are essential since most of the distinctive features to discriminate it from other objects are obvious in these two directions. Unfortunately, the original approach only gives rise to almost as many viewpoints in these two groups as the first group, which is far from enough to depict the blimp’s features.

Fig. 2 Viewpoint distribution for the blimp

(b) Test result relying on both EV and EEE events

Fig. 3 Viewpoints group division

With these divisions, each group of the viewpoints found by the two approaches can be employed to identify the physical nature of the EEE events.

Figures 4-6 compare the results for the original and the new approach including both the EV and EEE events. Including the EEE events introduces many more viewpoints into the second and third projections. The results show that the EEE events can provide more information on the objects irregular marginal features. As far as the blimp is concerned, numbers of viewpoints in the first group are equal. The common feature of this group is that they are all basically circles, which means that EEE events do not reflect the features of circles or at most show only as much information as the EV events. The other two groups are quite different.

Fig. 4 Blimp’s first group projections and silhouettes corresponding to the viewpoints in Fig. 2

(a) Results of original approach

(b) Results of new approach

Fig. 5 Blimp’s second group projections and silhouettes corresponding to the viewpoints in Fig. 2

(a) Results of original approach

(b) Results of new approach
Table 2 lists other viewpoint partitioning results for all five objects. The first column lists the result of the original method which only takes EV events into consideration while the second column shows results from the approach which identifies both EV and EEE events.

All the results show that the EEE events provide almost 30% of an object’s critical regions, which demonstrates the importance of EEE events in aspect graph theory.

5 Conclusions and Further Work

This paper presents a viewpoint space partitioning algorithm for EEE events and showed results for the partitioned space for various objects. The visualized partitioned space showed how it corresponds to the real case. Moreover, the results illustrated the strong link between EEE events and the object’s physical features. Matching of object results to expanded databases can achieve more accurate recognition.

The physical nature of the EEE events can be identified from the classification results. A better understanding of the EEE events will provide more information on the objects so the classification can improve 3-D object recognition.

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References

Overseas Experts and Students Celebrate New Year

Nearly one thousand Tsinghua foreign students from over 100 countries rang in the New Year on December 18, 2008 with an evening of memorable music and dance performances at Tsinghua’s Auditorium. Vice Chair of the Tsinghua University Council Shi Zongkai attended the event and extended New Year’s wishes to the students.

Meanwhile, more than 80 of Tsinghua’s overseas experts and their families gathered to celebrate New Year with Foreign Affairs staff and staff from different departments and schools on the evening of December 19, 2008.

In 2008, Tsinghua hosted more than 150 long-term overseas experts and over 760 short-term visit overseas scholars from 50 countries and regions including the United States, Britain, Russia, Germany, France, Australia, Japan, and Korea. The overseas experts play a significant role in teaching and research at Tsinghua University.

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