

# Humanoid-Eye Imaging System Model with Ability of Resolving Power Computing

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**Abstract.** This paper proposes an innovative imaging system model for human eyes with resolving power calculation, which is feasible in practice and stands on solid Physical background. The model, humanoid-eye imaging system (HIS), is constructed synthesizing an imaging component model and a photo-sensing component model based on relevant parts of human eyes. HIS integrates core features and working mechanism of human eyes and can also be regarded as simulation of various real digital imaging systems. According to criteria derived from wave optics and the theory of receptors, point resolving power for HIS is defined and its calculations are deduced as functions of specified parameters of HIS and variables of object points observed by HIS. Experiment with a camera as the application of HIS show that HIS is applicable and its resolving power calculation is precise in reality. Our work supply a novel method for the first time to efficiently connect real observing conditions with computer simulation for fields related to 3D meshes management.

**Keywords:** Humanoid-eye imaging system (HIS), resolving power, wave optics, receptor.

## 1 Introduction

The simplification of 3D objects recognition procedure through simplifying objects' 3D mesh model and managing multi-resolution model rendering is one of the methods to decrease the computation complexity of 3D objects recognition and accelerate the speed of such procedure, which gain increasing concern at present.

The last few years have seen many researchers' innovative and practical progress in this field [1, 2, 3]. No matter what approaches are employed in data management and simplification, within most of these methods there is a critical step to determine the extent to which the simplification should be stopped. When applied in reality, a typical way of such step is to assign a threshold which indicates the termination of 3D meshes simplification [4]. The threshold is typically expected to reflect real situations of certain observing systems such as human eyes, cameras and so on, especially for works related to practical implementation or simulation.

Unfortunately, current approaches for simplification threshold determination lack supports from definite background and theories of physics. Most of these approaches

are hardly beyond rough estimation: some appoint a value as the resolution threshold [5, 6] and such process is simple but nevertheless has little to do with real conditions; some methods take into account some features of imaging instruments and suggest view-point oriented threshold determination algorithm, but they merely guess the form of the formula or fit with data to infer the threshold [7, 8]; some eschew the difficulties of analysis of real conditions to attain the threshold through experiments empirically[9].

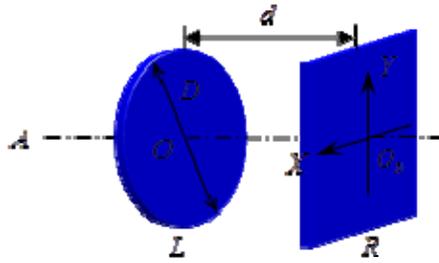
In perspective of acquiring a model close to reality and intensify the credibility of the results, it is in great need to set up an approach of objects' 3D meshes simplification threshold determination, which is supported by background of reality of physics and is suitable for practical implementation in computer simulation. Relied on these principles, this paper propose a physics model -- humanoid-eye imaging system (HIS), which integrates main features of geometric optics imaging and photo- sensing components of human eyes. By Rayleigh's law and theory of receptors, the definition and criteria of point-distinguishability (that is, whether object points can be distinguished by HIS) are presented and the angle resolving power (ARP) for HIS is defined. Given both of the criteria and the definition, we subsequently calculate the view-point-dependent resolving power for HIS (in angle and length) as functions of parameters of HIS and object points' variables such as distance between object points and HIS, azimuth of object points relative to HIS, deflection of object points and speeds of object points. For real implementations, the relationship between resolving powers by wave optics and theory of receptors are evaluated and a judgment is deduced to determine resolving power calculation formula for a specific imaging system. An experiment with a digital camera as the application of HIS is described and the results are showed to demonstrate the approach's credibility and reliability in practice.

The achievement of this paper supplies an innovative means, which is close to real conditions of human eyes and other imaging systems in practice, to determine the resolving power scale factor in multi-resolution 3D mesh management and rendering for graphics-related work (specifically, objects 3D recognition and virtual reality 3D models rendering, etc.).

## 2 Composition and Structure of HIS

By now, physiology has gained considerable knowledge of human vision, especially in the anatomical structure of human eyes and relevant procedure such as optics imaging and photo-sensing [10]. Results from this area show that the real physical structure of human eyes is complex. But in views of imaging principle and photo-sensing process, human eyes have substantial amounts of characteristics in common with various practical optics imaging systems.

Given the abstracted imaging and sensing simplified models, an imaging system, Humanoid-eye Imaging System, is then constructed as Figure 1 shows. This system is the physics equivalent model of the whole optical imaging and sensing system of human eyes. Parameters and components in Figure 1 are explicated as following.



**Fig. 1.** System diagram of HIS

- $L$  Imaging convergent lens (typically circular).
- $R$  Planar sensing arrays, parallel with  $L$ , with a receptors' density function  $\rho(x, y)$
- $O$  Optical center of  $L$ .
- $A$  Main optical axis of the system, vertical to  $L$  and  $R$  with  $A$  passing through  $O$  and  $O_r$ .
- $D$  Diameter of  $L$ .
- $d$  Distance between  $L$  and  $R$ .
- $T$  Exposure time of the system.

HIS strongly focuses on the fundamental features of imaging and sensing procedure of human eyes. The working principle of HIS can be divided into imaging procedure and sampling procedure. In imaging procedure, beam from the outside world is converged by  $L$  when passing to generate a clear image on planar sensing arrays. In sampling procedure, after exposure time  $T$ , each receptor on the plane transforms the luminous energy cast on it to independently output a pixel in electronic signal and all of these pixels together compose an image output.

### 3 Inferring of Resolving Power for HIS

#### 3.1 Criteria and Definition of ARP for HIS

The ability of imaging system to distinguish two object points is called the system's point resolving power (PRP). The minimum resolution angle, the minimum flair angle of two object points relative to the system when distinguishable, is often employed as the measurement of PRP and in this occasion, PRP is called angle resolving power (ARP).

According to nature of electromagnetic wave, accompanied with geometric optics are effects of wave optics, in which diffraction is a significant one. The diffraction causes image of a single object point on sensing plane to be a vague spot, Airy disk, with Airy disk's center as the ideal geometry image point. It is widely accepted that the necessary and sufficient conditions for two spots to be distinguished by an optical

lens is that the distance between two Airy disks' centers is greater than the radius of each Airy disk, which is the famous Rayleigh's law [11].

Theory of human eyes' receptors holds the viewpoint that when object points are sensible to a single receptor, there must be at least one receptor not stimulated by light between the two receptors, on which two object points' images cast separately, so as to ensure the two points are distinguishable on image. There are strong reasons behind this judgment: when image spots of two object points cast on two neighboring receptors, the system is unable to tell this situation from another one that an unique object point's image spot casts right on the boundary of the two receptors, which results in image the same as the previous situation.

Considering all described above, we define two object points distinguishable to HIS when they satisfy both following criteria.

Criterion of wave optics: The distance between centers of the two object points' image spot is not less than the larger radius of the two Airy disks.

Criterion of receptors: There is at least one un-stimulated receptor between the two receptors on which the two object points' image spots separately cast.

We define the ARP of HIS as the minimum angle of two object points relative to optical center of HIS when they are distinguishable and right satisfy one of the criterion: the distance between centers of the two object points' image spot is equal to the larger radius of the two Airy disks, or there is only one un-stimulated receptor between the two receptors where the two object points' image spots rest. By each of the criteria, two ARPs can be achieved and, naturally, we select the larger one as the actual ARP for HIS.

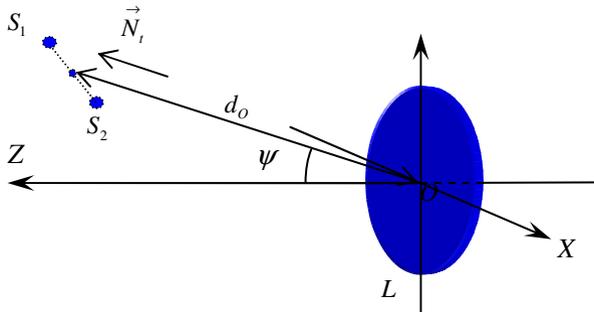
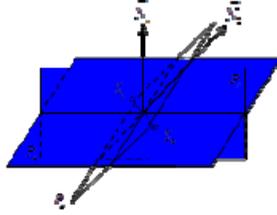


Fig. 2. Coordinator for observation of HIS

### 3.2 Calculation of ARP for HIS

Given object points' azimuth and speeds, parameters and principles of work of HIS (Fig.2 and Fig.3), two ARPs by both criteria are achieved. If positions and deflections of the points are available, the minimum distinguishable distance (MDD) of the points is also calculable and thus the threshold for distinguishability is established.

**ARP Calculation by Criterion of Wave Optics.** The criterion of wave optics requires that the distance between centers of two object points' Airy disks should be no less than the larger radius of the spots.



**Fig. 3.** Geometry relations of planes and normal vectors

By the criterion of wave optics, the ARP for HIS for static object points is the angle radius of the zero-order diffraction disk (it is easy to prove, using semi-wave-band method, that this angle radius is independent of azimuth when azimuth is not very large), that is:

$$\delta_{w01} = 1.22\lambda / D \quad (1)$$

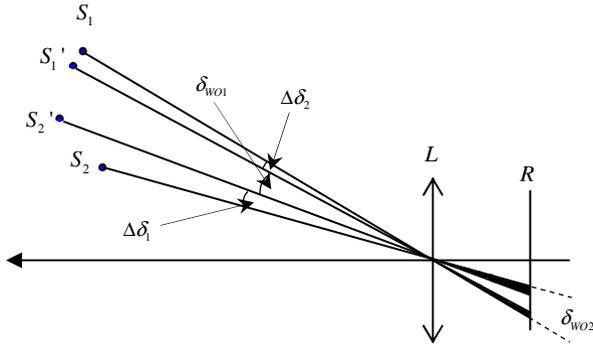
In dynamic case (the object is moving in space), however, we should consider the effects of the motion track in image on the angle of two points relative to optical center. During the exposure time, due to approaching motion, the two object points' image spots' angle relative to optical center decreases, as shown in Figure 4. The decreased angle is

$$\Delta\delta_{V_c} = \omega_c T = \frac{\|\vec{V}_c \cdot \vec{N}_\perp\| T}{d_o} \quad (2)$$

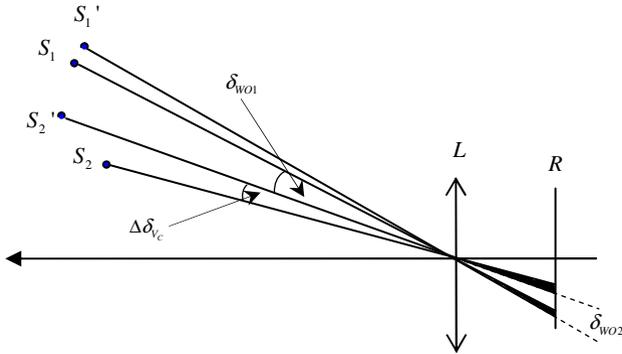
In this condition, ARP for HIS is modified to be

$$\delta_{w02} = \delta_{w01} + \Delta\delta_{V_c} = 1.22 \frac{\lambda}{D} + \frac{\|\vec{V}_c \cdot \vec{N}_\perp\| T}{d_o} \quad (3)$$

Given the positions of the two points, then the MDD along their segment is



(a) The increment of ARP is  $\Delta\delta_c = \Delta\delta_1 + \Delta\delta_2 = \omega_c T = \|\vec{V}_c \cdot \vec{N}_\perp\| T / d_o$



(b) The increment for ARP is  $\Delta\delta_{vc} = \omega_c T = \|\vec{V}_c \cdot \vec{N}_\perp\| T / d_o$

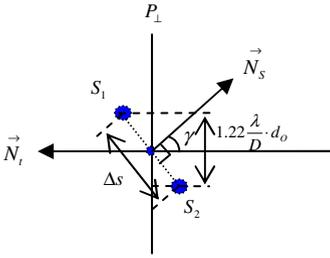
**Fig. 4.** Effects of approaching motion on image spots' angle

$$\Delta s(d_o, \vec{V}_c) = \frac{\delta_{w02} d_o}{\|\vec{N}_s \cdot \vec{N}_t\|} = \frac{(1.22 \frac{\lambda}{D} d_o + \|\vec{V}_c \cdot \vec{N}_\perp\| T)}{\|\vec{N}_s \cdot \vec{N}_t\|} \quad (4)$$

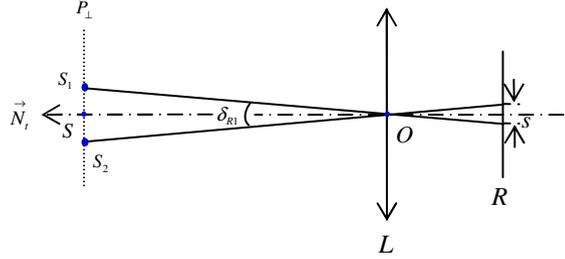
The item  $\|\vec{N}_s \cdot \vec{N}_t\|^{-1}$  is introduced as a modifier to compensate the deflection of two points relative to  $\vec{N}_t$ , as shown in Figure 5. It is obvious that the result is independent of any facts of planar sensing arrays.

**ARP Calculation by Criterion of Receptors.** The theory of receptors requirement for distinguishability of two object points is that there should be at least one unstimulated receptor between the two receptors on which the' image spots separately

cast. By this requirement, we evaluate effects of  $d_0$  (distance between object points and HIS),  $\Psi$  (azimuth of object points relative to HIS),  $\gamma$  (deflection of object points) and  $V_c$  (speeds of object points) upon the image cast on the planar sensing arrays one by one to calculate ARP and MDD for HIS.



**Fig. 5.** Compensator  $\|\vec{N}_s \cdot \vec{N}_t\|^{-1}$



**Fig. 6.** Illustration of effects of  $d_0$

i) Effect of  $d_0$

In order to investigate on the effect of  $d_0$  individually, we consider two static object points with  $\Psi = 0$ ,  $\gamma = 0$ . In such case,  $d_0$  and  $s$  together determine the ARP: when the angle of two point relative to optical center averagely covers right two receptor, the angle is ARP and it is easy to derive  $ARP\delta_{R1}$  form the Figure 6:

$$\delta_{R1} = 2l(0,0) / d = s / d_0 \quad (5)$$

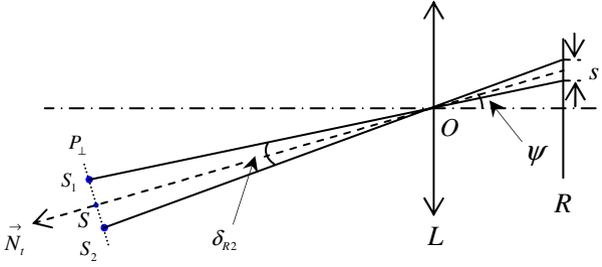
ii) Effect of  $\Psi$

When considering effects of azimuth  $\Psi$ , facts need focus are the density variation relative to that on planar sensing arrays' origin and the length covered by the angle. Given the azimuth  $\Psi$  of two points, the length  $s'$  on the planar sensing arrays covered by the angle is (as Figure 7 illustrates)

$$s' = \theta \cdot \frac{d}{\cos \psi} \cdot \frac{1}{\cos \psi} = \frac{s}{d_0} \cdot \frac{d}{\cos^2 \psi} \quad (6)$$

When length  $s'$  is equal to double of the one-dimensional length of the local receptors, the angle is ARP:

$$s' = 2l(\psi(x_o, y_o, z_o)) = 2l\left(-\frac{x_o}{z_o}, -\frac{y_o}{z_o}\right) \quad (7)$$



**Fig. 7.** Effects of azimuth  $\psi$

Here  $(x_o, y_o, z_o)$  is the position of the geometry center of the two points. ARP $\delta_{R2}$  is then presented as

$$\delta_{R2} = 2l(\psi(x_o, y_o, z_o)) \frac{\cos^2 \psi}{d} = 2l(S_1, S_2) \frac{\cos^2 \psi}{d} \quad (8)$$

iii) Effect of  $\vec{V}_c$

The effect of relative approaching speed  $\vec{V}_c$  is similar to that by the criterion of wave optics and the ARP $\delta_{R3}$  involving  $\vec{V}_c$  in dynamic situation is

$$\delta_{R3} = \Delta\delta_{V_c} + \delta_{P2} = \frac{\|\vec{V}_c \cdot \vec{N}_\perp\| T}{d_o} + 2l(\psi) \frac{\cos^2 \psi}{d} \quad (9)$$

### 3.3 Method for ARP Determination in Practice

Since the two criteria should be satisfied at the same time for HIS to distinguish two object points, both of the resolving powers by the two criteria should be calculated to find the actual resolving power for HIS. However, for most of real imaging systems, the statuses of the two criteria are not equal. Reviewing formulas for ARP and MDD calculations, we find that they are similar in structure:

$$\begin{cases} \delta_{wo} = 1.22 \frac{\lambda}{D} + \frac{\|\vec{V}_c \cdot \vec{N}_\perp\| T}{d_o} \\ \delta_R = 2l(\psi) \cdot \frac{\cos^2 \psi}{d} + \frac{\|\vec{V}_c \cdot \vec{N}_\perp\| T}{d_o} \end{cases} \quad (10)$$

$$\begin{cases} \Delta s_{wo} = (1.22 \frac{\lambda}{D} d_o + \|\vec{V}_c \cdot \vec{N}_\perp\| T) \cdot \|\vec{N}_s \cdot \vec{N}_i\|^{-1} \\ \Delta s_R = (2l(S_1, S_2) \frac{\cos^2 \psi}{d} d_o + \|\vec{V}_c \cdot \vec{N}_\perp\| T) \cdot \|\vec{N}_s \cdot \vec{N}_i\|^{-1} \end{cases} \quad (11)$$

Difference between formulas by the two criteria is the static item

$$J_{wo} = 1.22 \frac{\lambda}{D} d_o; J_R = 2l(P_1, P_2) \frac{\cos^2 \psi}{d} d_o$$

We define these two items as judgment of wave optics and judgment of receptors, which indicates resolution limit thresholds by the two criteria. Note that these judgments are determined by inherent feature parameters of HIS and by these judgments, resolving power of different applications of HIS can be assessed. For implementations in a specific imaging system, the comparison of the two judgments should be made first. Provided all outside conditions are the same, the set of formulas for ARP and MDD of which the judgment is larger is chosen as resolving power calculation for the system and the smaller one's relevant criterion will be automatically satisfied.

## 4 Experiments

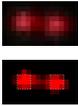
To prove the formula of APR and MDD for HIS, an experiment is carried out with a digital camera (Canon PowerShot A710 IS) as the practical implementation of HIS and two LEDs as object points.

Relied on parameters of the camera, we calculate the judgments of the system to get  $J_{wo} = 4.538 \times 10^{-5}$  and  $J_R = 4.05 \times 10^{-4}$ . The resolution limit threshold by theory of receptors far exceeds that by wave optics. Therefore the actual resolving power calculations for ARP and MDD are Eq. (8) and Eq. (9).

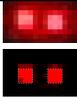
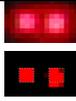
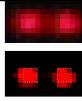
### 4.1 Procedure and Results

In the experiment, the effects of factors  $d_o$ ,  $\Psi$  and  $\gamma$  are proved individually for the static situation. For dynamic situation, the effect of  $d_o$  is tested and effects of  $\Psi$  and  $\gamma$  are evaluated together, with different speed Vc settings for each sampling data groups. The actual approach is to calculate MDD by given parameters, then set the distance of the two LEDs to be the predicted MDD and sample images to see whether the prediction will match the results. By setting an area of  $3 \times 3$  pixels as the equivalent receptor, two points are just distinguishable when the areas of  $3 \times 3$  pixels where LEDs' images are located are right separated by an equivalent receptor. Some adjustment is made on original images for a better localization of LEDs. Some of the results are showed in tables 1 to 4, including original images (upper ones in each image set) and adjusted images (lower ones in each image set).

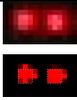
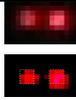
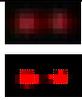
**Table 1.** Predicted static MDD and results for the effect of  $d_o$

$d_o / \text{mm}$	1500	3300	6300
$\Delta s / \text{mm}$	3.33	7.33	14.00
image			

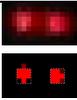
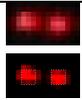
**Table 2.** Predicted static MDD and results for the effect of  $\gamma(d_0 = 1800\text{mm})$ 

$\gamma/^\circ$	10	40	80
$\Delta s/\text{mm}$	4.06	5.22	23.04
image			

**Table 3.** Predicted static MDD and results for the effect of  $\gamma(d_0 = 3000\text{mm})$ 

$\gamma/^\circ$	10	40	80
$\Delta s/\text{mm}$	6.77	8.70	38.39
image			

**Table 4.** Predicted static MDD and results for the effect of  $\Psi$ 

$\Psi/^\circ$	0	+10	+20
$\Delta s/\text{mm}$	8.00	7.76	7.06
image			

## 4.2 Discussions

In the static part, there are 27 out of 35 (approximately 80%) images strictly satisfying the definition of just distinguishable situation defined in section 4.2. For the rest 20 percent, the error is only one pixel. Allowing for the fact that CCD samples image discretely to cause an error of a pixel for two LEDs (as discussed below), we treat these 20 percent's image falling within the error threshold and regard the prediction made by MDD formula as congruous with the whole static results. In the dynamic part, 5 out of 13 images strictly match the just distinguishable situation and for the rest images, 7 out of 8 fall in the error threshold of one pixel. We consider the dynamic results agreeable to the MDD calculation with only one image's violation.

The main source of error in the experiment comes from discretely imaging of CCD. Rather than an ideal point-photo source, the structure of LED is actually a light-emitting plane and other factors such as diffraction and diffusion also contribute to the

non-ideality. The error is at most half a pixel in situation that one edge of a LED's image falls around the center of a pixel and two LEDs' images will form an error of one pixel together at most.

Reviewing all the results, the experiment demonstrates nearly 98% images in favor of the MDD prediction and strongly supports the formula for both of ARP and MDD we achieve in chapter 3.

## 5 Conclusions

This paper abstracts two equivalent models of human eyes' imaging and photo-sensing parts, and then constructs HIS, an innovative imaging system which integrates main features of human eyes and many other practical imaging systems. We derive criteria of point-distinguishability for HIS and define its resolving power by Rayleigh's law and theory of receptors, which restrict the resolving power of HIS. By these criteria and definition, we propose two sets of formulas for ARP and MDD calculation, involving feature parameters of HIS and object points' variables. The experiment result shows that predictions of the resolving power calculation formulas for HIS are congruous with reality, with nearly 98% correct ratio. We have successfully simulated HIS and integrated the formulas of its resolving power in a multi-resolution object 3D models recognition computer program. The results provide supports for outstanding efficiency and applicability of HIS and relevant resolving power calculation in 3D mesh management. Future work includes evaluation of other factors related to resolving power of HIS and further application in computer.

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