

# The Electronic Countermeasures Optimization Based on PCA and Multiple Regression Analysis

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**Abstract.** Looking for the best solution for Confrontation is crucial for the effect of electronic countermeasures. In this paper, we put forward the concept about the optimal laying space [1], analyze and evaluate the simulation results in different countering situation base on the maritime infrared electronic countermeasures simulation platform [2]. Then, here comes up with a way to get the optimal laying space based on PCA [3], which effectively solved the problem of strategy optimization in electronic countermeasures. Also, the paper builds a model to evaluate the probability of countering success by using the multiple regression analysis [4], and provides the probability parameter for the selected best countering strategy. The testing results match properly with the results from the simulation platform, indicating that this method is able to find the best strategy for electronic countermeasures.

**Keywords:** electronic countermeasures, strategy optimization, PCA, multiple regression.

## 1 Introduction

The electronic countermeasures system generally includes two entities—attacking side and defensive side. Under certain circumstances, various environmental factors such as wind speed, wind direction are regarded as an entity, namely environmental entity. Actually, the research on electronic countermeasures is to simulate the interactions between each entity, formulate countering strategy and effective evaluation for the strategy. To sum up, the relationship among the entities in electronic countermeasures can be summarized as the following diagram (Fig.1):

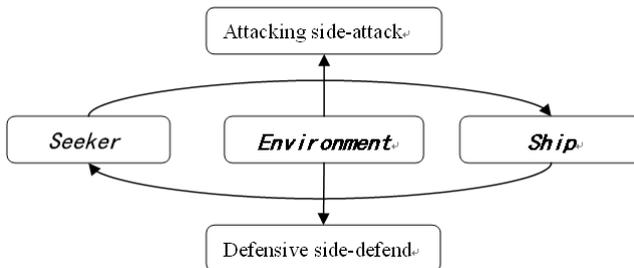
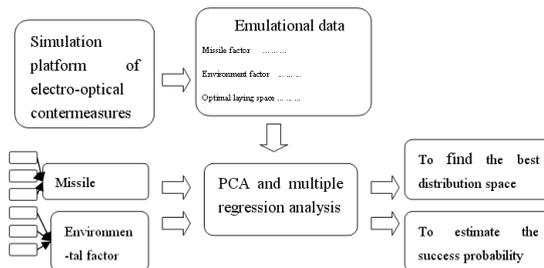


Fig. 1. Electronic countermeasures system

In recent years, electronic countermeasures system has become a focal point of research domestically and internationally. There is some progress in electronic countermeasures modeling, for example, the electronic countermeasures simulation system based on HLA[5] in literature 1 and the electronic countermeasures simulation system using Multigen Creator to model the environment in literature 2. All the achievements mentioned above have provided good solutions for modeling the electronic countermeasures system.

With the solution of modeling the electronic countermeasures system, the key points and the difficulties of the research in electronic countermeasures remains in how to formulate countering strategy and effective evaluation for the strategy. Previously, the study in this area is almost restricted in the theory analysis according to the principle of confrontation. While the appropriate performance assessment methods are absent, actually, formulation of the strategy was often influenced by many subjective factors. In addition, the so-called decision-making "optimization" work was actually a kind of "pre-optimization", and that was choosing a better one among several options prior to the confrontation as a practical countering strategy. Although this commonly used method has already been well developed, the flexibility was greatly restricted.

Meanwhile, in the previous study domestically and internationally, the confrontation "decision-making" is often aiming at finding the way to launch the jamming shells, that is, how to send jamming shells (including the type, quantity, launch location of jamming shells and so on) to make the best interference. Optimization in the decision-making overlooks the impact of other variables, that is, no matter what the measured parameters of attacking side (seeker) such as distance, direction, and all kinds of environmental factors such as wind direction, wind speed are, only one type of way to launch the jamming shells is used, or one in several ways is selected. In this case, the effective evaluation of the method is recapitulative, and cannot be carried out in a particular environment.



**Fig. 2.** The model to evaluate the success probability of confrontation

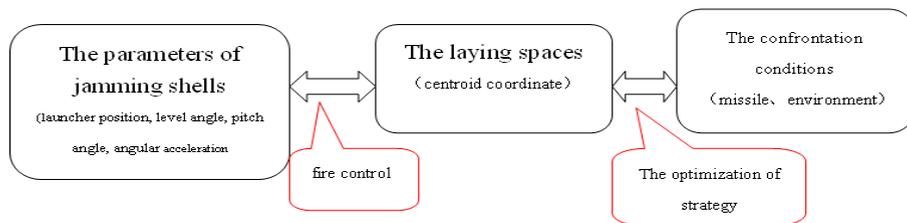
Taking infrared electronic countermeasures at sea as specific research background, pointed to the optimization of electronic countermeasures strategy, this paper introduces the concepts of the optimal laying space. The paper has taken into account all kinds of factors which have a great impact on the result of the confrontations, including seeker factors and environmental factors. In short, it is under specific conditions that the counterplot is optimized "in real time". In addition, this study

combines simulation tests with theoretical analysis. According to the existing data from the simulation platform and aiming at the seeker and environmental factors, this study raises a model based on PCA (Principal Component Analysis) method for solving the optimal laying space. By using the multiple regression analysis method, we set up a model to evaluate the success probability of confrontation, as Fig. 2. Such methods and results have more practical value and guiding significance than previous research.

## 2 The Optimal Jamming Shells Laying Space Proposed

### 2.1 The Definition of Optimal Laying Space

In electronic countermeasures, firing the jamming shells is one of the effective combat methods. The core problem of confrontation strategy optimization is to optimize the position where the jamming shells are distributed. In the actual confrontation, the position where the jamming shells are distributed is decided by many parameters such as launcher position, level angle, pitch angle, angular acceleration. This is a complex problem of fire control. The previous studies often optimized the strategies which based on the parameters of jamming shells. Because of the high complexity, it was difficult to optimize the strategies “objectively”. Introducing the concept of laying space, we divide and conquer the research on strategy optimization and the problem of fire control. In this way, it is effective in reducing the decision-making complexity to use the parameters of laying space instead of the parameters of jamming shells, the model like Fig. 3.



**Fig. 3.** The procession of the optimization of strategy

The jamming shells laying space is defined as follows: the space where the jamming shells were distributed is called the jamming shells laying space;

1) For the same types of jamming shells, suppose that the volume and shape of every laying space is equal and every laying space is a cuboid whose length, width and height are fixed.

2) Suppose that the base plane of the laying space parallels to the horizontal plane. And the height of the laying space is perpendicular to the horizontal plane. And one plane of the laying space is perpendicular to the seekers and ships connection.

3) In the coordinate whose origin is defensive side (ship) and X-Y axis is horizontal plane, the rectangular coordinate  $(X, Y, Z)$  and the polar coordinate  $(r, \theta, \varphi)$  can be used to describe the centroid of the laying space absolutely.

4) Determine the number of laying spaces due to the specific environment of electronic confrontation and combat experience. And according to the main effect each laying space takes, choose several laying spaces whose value of optimization is larger. Through simulation, sample and enumerate in the spaces within a certain range to optimize the laying spaces. Under certain confrontation conditions, the laying space with the highest confrontation success rate would be the optimal laying space.

## 2.2 The Acquisition of Data Samples

In this study, the data samples were acquired through the maritime infrared electronic countermeasures simulation platform. This platform can simulate the real process of maritime infrared confrontation and give accurate confrontation results in different situation (such as seeker parameters, environment parameters etc.).

For each infrared electronic countermeasures experiment, the main parameters determining the optimal laying spaces and confrontation success rates are wind direction  $\theta_1$ , wind speed  $v_1$ , seeker distance  $l_2$ , seeker direction  $\theta_2$ , seekers instantaneous velocity  $v_2$ . Meanwhile, considering that the measurement of the above five parameters exists errors in the actual combat, the random noise is added to the above known variables in a single experiment for the authenticity of simulation. Then sample and enumerate the laying spaces through the simulation platform. After several experiments, we can get the optimal laying space and the highest confrontation success probability, that is, the following data form.

**Table 1.** Data form

$\theta_1, v_1, \theta_2, v_2, l_2$	The optimal laying space	Success probability p
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According to several groups of the confrontation condition parameters  $\theta_1, v_1, \theta_2, v_2, l_2$ , the optimal laying spaces with success probability are acquired. Because the optimal laying space is acquired through sampling and enumerating, the collection of the laying spaces is an enumerable collection. Everyone in the collection is an optimal laying space corresponding to several groups of confrontation condition parameter.

## 2.3 Solving Optimal Laying Space Based on PCA

The progress to acquire the optimal laying space through the simulation platform under a confrontation condition is not real-time. And the confrontation condition parameters  $\theta_1, v_1, \theta_2, v_2, l_2$  are not enumerable. The strategy-making in actual combat need clarify the relationship between the confrontation condition parameters and the choice of the optimal laying space in real time. In this research, each component of the inputting

vector  $(\theta_1, v_1, \theta_2, v_2, l_2)$  has a different effect on choice of the optimal laying space. And we cannot set up the weight of each component subjectively. The PCA can help us to accomplish this mission well.

PCA (Principal Component Analysis-PCA), is also known as the main component analysis, sometimes also is known as eigenvector analysis. As the name suggests, PCA is the main part of the data extracted, which describe the characteristics of objects. In fact, from the point of view of geometric transformations, the basic idea of PCA is to find an optimal subspace. When the high dimensional data in the subspace is for projection, the income component has the greatest variance. At the same time, when using the new components to rebuild the origin data in the subspace, the approximation effect is the best at the meaning of the minimum mean square error.

- Data training Process

For inputting N-dimensional vector  $\vec{x}$  (here,  $N=5$ ,  $\vec{x} = (\theta_1, v_1, \theta_2, v_2, l_2)$ ), The purpose of PCA is to find a orthogonal transformation matrix  $W^T = (w_1, w_2, \dots, w_M)$ , linear transforms X to Y so that every componnt of Y is linearly independent. The transformation is known as K-L Transform. Ther are o lot of ways in its numerical calculation, the following gives the method used in our study:

First, for each value of the laying spaces, get all its corresponding parameter vectors. Suppose the number of vectors is L, they are  $X_1, X_2, \dots, X_L$ ,  $X_i = (\theta_1^{(i)}, v_1^{(i)}, \theta_2^{(i)}, v_2^{(i)}, l_2^{(i)})$ . These vectors are formed as:

$$S = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_L \end{pmatrix}_{L \times 5} \quad (1)$$

All vectors here have been standardized,  $X_i = X_i - \bar{X}$ , Correlation matrix is solved

from the following formula:  $R = S^T \cdot S$

Find the eigenvalues and eigenvectors of the matrix. In accordance with the descending order of eigenvalues, we get eigenvalues  $\lambda_1, \lambda_2, \dots, \lambda_5$  corresponding with eigenvectors  $\mu_1, \mu_2, \dots, \mu_5$ . These five 5-dimensional eigenvectors  $\mu_1, \mu_2, \dots, \mu_5$  can be a full description of the laying space. Meanwhile, corresponding eigenvalues  $\lambda_1, \lambda_2, \dots, \lambda_5$  express the amount of information contained in  $\mu_1, \mu_2, \dots, \mu_5$ . Of course the more eigenvectors used, the more fully they describe. This study choose eigenvectors as the following rules:

$$r(k) = \frac{\sum_{i=1}^k \lambda_i}{\sum_{i=1}^L \lambda_i} \quad (2)$$

k is the number of eigenvectors, and L is the total number of eigenvectors namely  $L=5$ . Increase k from 1 until  $r(k) \geq 0.99$  so that we get 99% information. For all values of the laying spaces, describe them by the following tow parameters:  $(k, \mu_1, \mu_2, \dots, \mu_k)$ .

- Data Reconstruction Process

The Eigenvector Extraction above accomplish data training process, it needs not to be processed in actual combat. While the Eigenvector matching is a data reconstruction process to find optimal laying space. Match inputting vector  $\vec{x} = (\theta_1, v_1, \theta_2, v_2, l_2)$  to every laying space's eigenvector, the one with the best match is the optimal laying space under certain condition.

Describe the matching performance with the distance defined as follows:

$$E(x)^j = \min_{k=1, \dots, L} \left\| \sum_{i=1}^{M_j} \mu_{ci}^T (x - x_k) \mu_{ci} - (x - x_k) \right\|^2 \quad (3)$$

Here, j is subscript taking over all the values of the laying space,  $M_j$  is the corresponding number of eigenvectors,  $x$  is a sample vector in the laying space. For the laying space j,  $E(x)^j$  actually is the minimum of the difference between projections of  $\vec{x} = (\theta_1, v_1, \theta_2, v_2, l_2)$  and vectors corresponding to the laying space j in the feature subset space of space j.

In this way, select the least value in  $\{E(x)^j\}$ , the corresponding laying space is the optimallaying space. The space with the second least value is the second optimal laying space, etc.

### 3 Multiple Regression Analysis for Success Probability of Confrontation

Under the condition with parameters  $\vec{x} = (\theta_1, v_1, \theta_2, v_2, l_2)$ , the relationship among  $\vec{x} = (\theta_1, v_1, \theta_2, v_2, l_2)$ , optimal laying space  $\Omega$  and corresponding success probability p are shown as follows:

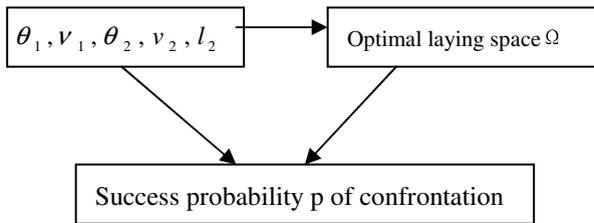


Fig. 4. The structural relationship

For a single electronic countermeasures experiment:

$$p(\text{success of confrontation} | x, \Omega) = p(\text{success of confrontation} | x)$$

Accordingly, the success probability only concerns known parameters and is independent of optimal laying space with certain  $x = (\theta_1, v_1, \theta_2, v_2, l_2)$ . The success probability is kind of function of  $x = (\theta_1, v_1, \theta_2, v_2, l_2)$ , namely  $p = f(\vec{x})$ . Apparently this function is not a simple linear function.

Regression analysis is a collective name for techniques for the modeling and analysis of numerical data consisting of values of a dependent variable and of one or more independent variables. The dependent variable in the regression equation is modeled as a function of the independent variables, corresponding parameters, and an error term. Generally linear regression analysis is a basic method, when the model function is not linear, it's always converted into linear regression with mathematics. In this study we take the following ways:

For  $p = f(\vec{x}) = f(\theta_1, v_1, \theta_2, v_2, l_2)$ , suppose the existence of a linear function  $f'$ , so that :

$p = f(\vec{x}) = f'(\Phi(\vec{x}))$ ,  $\Phi(\vec{x})$  is assumed by analyzing and processing the sample data. Here are 4 ways to choose  $\Phi(\vec{x})$  :

- $\Phi_1(\vec{x})$  by analyzing the sample data :

Ignore the coupling relationship in factors (except  $\theta_2$  and  $\theta_1$ ), suppose:

$$p = f'(\Phi_1(\vec{x})) = f'(\phi_1(\theta_1), \phi_2(v_1), \phi_3(\theta_2, \theta_1), \phi_4(v_2), \phi_5(l_2)) \quad (4)$$

Because  $\theta_2$  and  $\theta_1$  are respectively seeker direction and wind direction, the coupling relationship between them is apparently. Considering the relativity of direction, may wish to set  $\phi_3(\theta_2, \theta_1) = \phi_3(\theta_2 - \theta_1)$ , then

$$p = f'(\Phi_1(\vec{x})) = f'(\phi_1(\theta_1), \phi_2(v_1), \phi_3(\theta_2 - \theta_1), \phi_4(v_2), \phi_5(l_2)) \quad (5)$$

$\phi_1, \phi_2, \phi_3, \phi_4, \phi_5$  Is simple function by assured by analysing and fitting data, often is the form of power function or polynomial function;

- $\Phi_2(\vec{x})$  by considering physical relationship in parameters:

Through analysing the physical progress of confrontation, find the parameters or simple function of parameters which have great impact on success of confrontation.

These parameters or functions form  $\Phi_2(\vec{x})$  ;

- $\Phi_3(\vec{x})$  by using kernel-regression. Assure  $\Phi_3(\vec{x})$  comparing different mixed kernel function :

Suppose polynomial kernel function  $K_1(x, x_i) = (x \cdot x_i + k)^d$ , Gaussian kernel

function  $K_2(x, x_i) = \exp(-\frac{\|x - x_i\|^2}{2\sigma^2})$ , form mixed kernel function

$$K(x, x_i) = rK_1(x, x_i) + (1-r)K_2(x, x_i), \quad r \in (0, 1)$$

- $\Phi_4(\bar{x})$  by comprehensive using above 3 ways and increase terms of function:

After choosing the function  $\Phi$ , and noting that  $\bar{x}' = \Phi(\bar{x})$ , so  $p = f'(X')$ . The form

of multiple regression is: 
$$\begin{cases} p_i = f(\bar{x}'_i) = \beta_0 + \beta_1 x'_{i1} + \dots + \beta_m x'_{im} + \varepsilon \\ \varepsilon \sim N(0, \sigma^2), i = 1, \dots, n \\ \bar{x}'_i = \Phi(\bar{x}_i) \end{cases}$$

Noting that 
$$X' = \begin{bmatrix} 1 & x'_{11} & \dots & x'_{1m} \\ \vdots & \vdots & & \vdots \\ 1 & x'_{n1} & \dots & x'_{nm} \end{bmatrix}, \quad P = \begin{bmatrix} p_1 \\ \vdots \\ p_n \end{bmatrix}, \quad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{bmatrix},$$

$$\bar{\beta} = [\beta_0, \beta_1, \dots, \beta_m]^T$$

The formula above is 
$$\begin{cases} P = X' \beta + \varepsilon \\ \varepsilon \sim N(0, \sigma^2 I) \end{cases}$$
. The sum of squares error of the above data is

$$Q(\beta) = \sum_{i=1}^n \varepsilon_i^2 = (P - X' \beta)^T (P - X' \beta)$$
. Using the necessary condition for extremism,

$$\frac{\partial Q}{\partial \beta_j} = 0 (j = 0, 1, \dots, m)$$
 Find  $\beta$  so as to minimize  $Q(\beta)$ , equations

are:  $X^T (P - X' \beta) = 0$ . The solution of equations is  $\bar{\beta} = (X^T X)^{-1} X^T P$ .

This is the Least squares estimation of  $\beta$ , which is also the result of regression analysis. For parameters  $\bar{x} = (\theta_1, v_1, \theta_2, v_2, l_2)$ , the estimated success probability is  $p = f'(\bar{x}') = \bar{x}' \cdot \bar{\beta} = \Phi(\bar{x}) \cdot \bar{\beta}$ .

### 4 Experiment and Conclusion

This paper solve the optimal laying space by using algorithm based on PCA. Analyzing the sample data from maritime infrared electronic countermeasures simulation platform. With 100 testing data—100 groups of  $\bar{x} = (\theta_1, v_1, \theta_2, v_2, l_2)$  randomly generated, get the results by both PCA algorithm and simulation. Compare two results of each  $\bar{x}$ , the match probability is 58%.

We use MAE (the average absolute error) to assess the estimated model set up by multiple regression analysis, MAE is defined as follows:  $MAE = \frac{\sum_i |p_i^{(t)} - p_i|}{n}$ .

$n$  is the amount of testing data,  $p_i^{(t)}$  is the success probability acquired from the simulation platform,  $p_i$  is the success probability calculated by the multiple regression model.

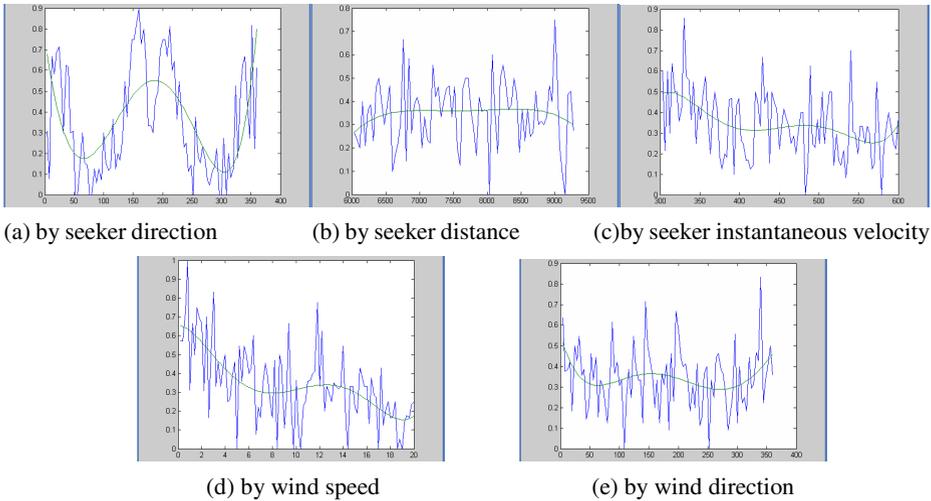
The following table shows MAE of the multiple regression model with different ways to make  $\Phi$ , we can compare them:

**Table 2.** MAE under different  $\Phi$

	$\Phi_1(\vec{x})$	$\Phi_2(\vec{x})$	$\Phi_3(\vec{x})$	$\Phi_4(\vec{x})$
MAE	0.2159	0.2435	0.2397	0.1664

In 4 models above, model 1, model 2 is of much simpleness. Because  $\Phi_1(\vec{x})$  and  $\Phi_2(\vec{x})$  is combination of simple functions, it is convenient to show the impact of each component of  $\vec{x}$  on the result of confrontation, which can guide the actual combat in revese; model 3 can be applied to cases when physical relationship is not clear, which is more universal; model 4 has best performance in this study about maritime infrared electronic countermeasures, which has a complicated  $\Phi$  function.

Take  $\Phi_1(\vec{x})$  as an example, for  $p = f(\vec{x}) = f_1'(\Phi_1(\vec{x}))$ ,  $\frac{\partial f}{\partial v_1} = -0.011v_1$ . The Fig. 4 shows wind direction, wind speed, seeker distance, seeker direction and seekers instantaneous velocity – varying success probability:



**Fig. 5.** The relationship between different parameters and success probability

As can see from figure, trends of the success probability from simulation platform is inoculated with the result of regression model.

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